

What rare K decays can tell about the MSSM

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Abstract. Supersymmetric contributions to the theoretically clean $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$ decays are briefly reviewed. Particular emphasis is laid on the information one could get on the MSSM flavor sector from a combined study of the four modes.

PACS. 12.60.Jv Supersymmetric models – 13.20.Eb Decays of K mesons

1 Introduction

The FCNC-induced decays, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$, are very suppressed in the Standard Model (SM), where they can be predicted very accurately. Therefore, these modes are ideal for probing possible New Physics effects[1]. In the present talk, the signatures of supersymmetry, in its simplest realization as the MSSM, are reviewed.

2 Rare K decays in the Standard Model

In the SM, the electroweak processes driving the rare K decays are the W box, Z and γ penguins[2], see Fig.1a. In this section, the excellent theoretical control reached on these contributions is summarized briefly.

The $K \rightarrow \pi \nu \bar{\nu}$ decays in the SM: The t -quark contribution to the Wilson coefficient of the dimension-six FCNC operator $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ is known at NLO[2], while the c -quark one has recently been obtained at NNLO[3]. The matrix-elements for this operator can be extracted from $K_{\ell 3}$ decays, including NLO isospin corrections[4]. For $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, residual c -quark effects from dimension-8 operators, along with long distance u -quark contributions, have also been computed [5]. For $K_L \rightarrow \pi^0 \nu \bar{\nu}$, the indirect CP-violating contribution (ICPV), $K_L \xrightarrow{\epsilon} K_1 \rightarrow \pi^0 \nu \bar{\nu}$, is of about 1%[6], and the CP-conserving one is less than 0.01%[7]. Altogether, the SM predictions are

$$\begin{aligned} \mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} &= (2.49 \pm 0.39) \cdot 10^{-11}, \\ \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} &= (7.83 \pm 0.82) \cdot 10^{-11}. \end{aligned}$$

The error on $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is mainly parametric, i.e. dominated by $\text{Im } \lambda_t$, $\lambda_t \equiv V_{ts}^* V_{td}$. For $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, which receives a significant c -quark contribution, the total error could be reduced with a better knowledge of m_c and through a lattice study of higher-dimensional operators[8].

The $K_L \rightarrow \pi^0 \ell^+ \ell^-$ decays in the SM: The situation is more involved because there are a priori three competing processes.

First, the t and c -quark contributions, known at NLO[2], generate both the dimension-six vector and axial-vector operators:

$$\mathcal{H}_{eff} = y_{7V} (\bar{s}d)_V (\bar{\ell}\ell)_V + y_{7A} (\bar{s}d)_V (\bar{\ell}\ell)_A.$$

The former produces the $\ell^+ \ell^-$ pair in a 1^{--} state, the latter in both 1^{++} and 0^{-+} states.

Secondly, the ICPV contribution is related to $K_S \rightarrow \pi^0 \ell^+ \ell^-$, which is dominated by the Chiral Perturbation Theory (ChPT) counterterm a_S [9]. NA48 measurements give $|a_S| = 1.2 \pm 0.2$ [10]. Producing $\ell^+ \ell^-$ in a 1^{--} state, it interferes with the $(\bar{s}d)_V (\bar{\ell}\ell)_V$ contribution, arguably constructively[11,12]. This sign could also be fixed experimentally from A_{FB}^μ , the integrated forward-backward, or muon-energy asymmetry[13].

The final piece is the CP-conserving two-photon contribution, which produces the lepton pair in either a helicity-suppressed 0^{++} or phase-space suppressed 2^{++} state. The LO corresponds to the finite two-loop process $K_L \rightarrow \pi^0 P^+ P^- \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 \ell^+ \ell^-$, $P = \pi, K$, exactly predicted by ChPT, and produces only 0^{++} states. Higher order corrections are estimated using experimental data on $K_L \rightarrow \pi^0 \gamma \gamma$ for both the 0^{++} and 2^{++} contributions[11,14].

Altogether, the predicted rates are

$$\begin{aligned} \mathcal{B}(K_L \rightarrow \pi^0 e^+ e^-)_{\text{SM}} &= 3.54_{-0.85}^{+0.98} (1.56_{-0.49}^{+0.62}) \cdot 10^{-11}, \\ \mathcal{B}(K_L \rightarrow \pi^0 \mu^+ \mu^-)_{\text{SM}} &= 1.41_{-0.26}^{+0.28} (0.95_{-0.21}^{+0.22}) \cdot 10^{-11}, \end{aligned}$$

for constructive (destructive) interference. The errors are detailed in [11,13,14], and are currently dominated by the one on the $K_S \rightarrow \pi^0 \ell^+ \ell^-$ rate measurements.

3 Rare K decays and supersymmetry

Even though the minimal supersymmetrization of the SM requires one super-partner for each SM particle

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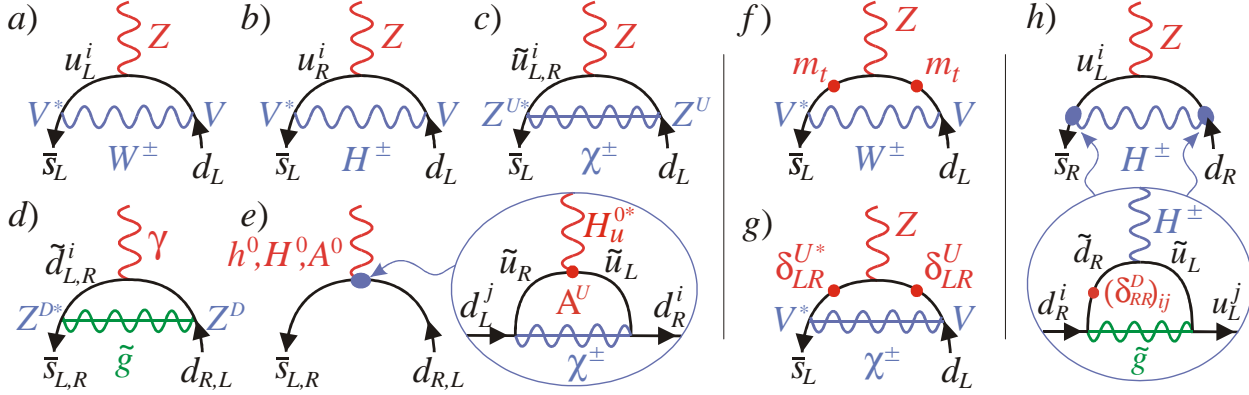


Fig. 1. *a – e)* Dominant MSSM contributions to rare K decays. *f – g)* Dominant sources of $SU(2)_L$ -breaking in the Z -penguin. *h)* Schematic representation of the H^\pm contribution to the Z -penguin at large $\tan\beta$.

(and two Higgs doublets), it is very constrained and involves only a few free parameters. However, SUSY must be broken, and the precise mechanism still eludes us. Therefore, in practice, an effective description is adopted, introducing all possible explicit soft-breaking terms allowed by the gauge symmetries. In the squark sector, there are LL and RR mass-terms and trilinear couplings giving rise to LR mass-terms after the Higgses acquire their VEV's, $\langle H_{u,d}^0 \rangle = v_{u,d}$:

$$\begin{aligned}\mathcal{L}_{soft}^{LL,RR} &= -\tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} - \tilde{U} \mathbf{m}_U^2 \tilde{U}^\dagger - \tilde{D} \mathbf{m}_D^2 \tilde{D}^\dagger, \\ \mathcal{L}_{soft}^{LR} &= -\tilde{U} \mathbf{A}^U \tilde{Q} H_u + \tilde{D} \mathbf{A}^D \tilde{Q} H_d,\end{aligned}$$

with $\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)^T$, $\tilde{U} = \tilde{u}_R^\dagger$, $\tilde{D} = \tilde{d}_R^\dagger$. Obviously, $\mathbf{m}_{Q,U,D}^2$ and $\mathbf{A}^{U,D}$, which are 3×3 matrices in flavor-space, generate a very rich flavor-breaking sector as squark mass eigenstates can differ substantially from their gauge eigenstates.

What to expect from SUSY in rare K decays: In the SM, the Z -penguin is the dominant contribution, and is tuned by λ_t (Fig.1a). The four MSSM corrections depicted in Figs.1b – e (together with box diagrams), represent the dominant corrections, and are thus the only MSSM effects for which rare K decays can be sensitive probes. Let us briefly describe each of them. First, there is the charged Higgs contribution to the Z -penguin (Fig.1b), which is, at moderate $\tan\beta = v_u/v_d$, aligned with the SM one ($\sim \lambda_t$). Then, there is the supersymmetrized version of Figs.1a – b, with charginos – up-squarks in place of W^\pm/H^\pm – up-squarks in the loop (Fig.1c), and which is sensitive to the mixings among the six up-squarks (Z^U), a priori not aligned with the CKM mixings. Another purely supersymmetric contribution, relevant only for charged lepton modes, is the gluino electromagnetic penguin (Fig.1d), sensitive to down-squark mixings (Z^D). The last class of effects consists of neutral Higgs FCNC (Fig.1e), and arises at large $\tan\beta \approx 50$. Indeed, the 2HDM-II structure of the Higgs couplings to quarks, required by SUSY, is not preserved beyond leading order due to \mathcal{L}_{soft} , and the “wrong Higgs”, H_u , gets coupled to down-type quarks, $\mathcal{L}_{eff} \supset \tilde{d}_R^j Y_d^{ik} (H_u^0 + \epsilon Y_u^\dagger Y_u H_u^{0\dagger})^{kj} \tilde{d}_L^i$ [15]. Clearly, once the Higgses acquire

their VEV's, there is a mismatch between quark mass eigenstates and Higgs couplings; both are no longer diagonalized simultaneously and Higgs FCNC are generated[16].

Bottom-up approach and Minimal Flavor Violation: There are too many parameters in \mathcal{L}_{soft} to have any hope to fix them all from rare K decays. At the same time, however, the observed suppression of FCNC transitions and CP-violating phenomena seem to indicate that only small departures with respect to the SM are possible. Therefore, one starts from a lowest-order basis in which the flavor-breakings due to $\mathbf{m}_{Q,U,D}^2$ and $\mathbf{A}^{U,D}$ are minimal. This can take the form of $mSUGRA$, alignment of squarks with quarks or the Minimal Flavor Violation hypothesis (MFV). In a second stage, one probes the possible signatures of departures from this minimal setting. The goal being, ultimately, to constrain SUSY-breaking models, which imply specific soft-breaking structures. At that stage, information from rare K decays, colliders and B -physics must of course be combined.

Here we adopt MFV as the lowest order basis, i.e. we impose that the SM Yukawas $\mathbf{Y}_{u,d}$ are the only sources of flavor-breaking[17]. In practice, this means that \mathcal{L}_{soft} terms can be expanded as $(a_i, b_i \sim O(1))$, and A_0, m_0 set the supersymmetry-breaking scale)

$$\begin{aligned}\mathbf{m}_Q^2 &= m_0^2 (a_1 \mathbf{1} + b_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \\ &\quad + b_3 (\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d)), \\ \mathbf{m}_U^2 &= m_0^2 (a_2 \mathbf{1} + b_4 \mathbf{Y}_u \mathbf{Y}_u^\dagger), \\ \mathbf{A}^U &= A_0 \mathbf{Y}_u (a_4 \mathbf{1} + b_6 \mathbf{Y}_d^\dagger \mathbf{Y}_d),\end{aligned}$$

and similarly for \mathbf{m}_D^2 and \mathbf{A}^D , such that all FCNC's and CP-violation are still essentially tuned by the CKM matrix. For example, the dominant contributions to the Z -penguin are those breaking the $SU(2)_L$ gauge-symmetry[18,19]. In the SM, this breaking is achieved through a double top-quark mass insertion (Fig.1f). Similarly, in the MSSM, it is the double $\tilde{t}_L - \tilde{t}_R$ mixing via the \mathbf{A}^U trilinear terms which plays the dominant role (Fig.1g in the sCKM basis)[20]. Within MFV, this gives a factor $m_t^2 \lambda_t |a_4 - \cot\beta \mu^*|^2$ [21], still enhanced by m_t^2 and tuned by λ_t .

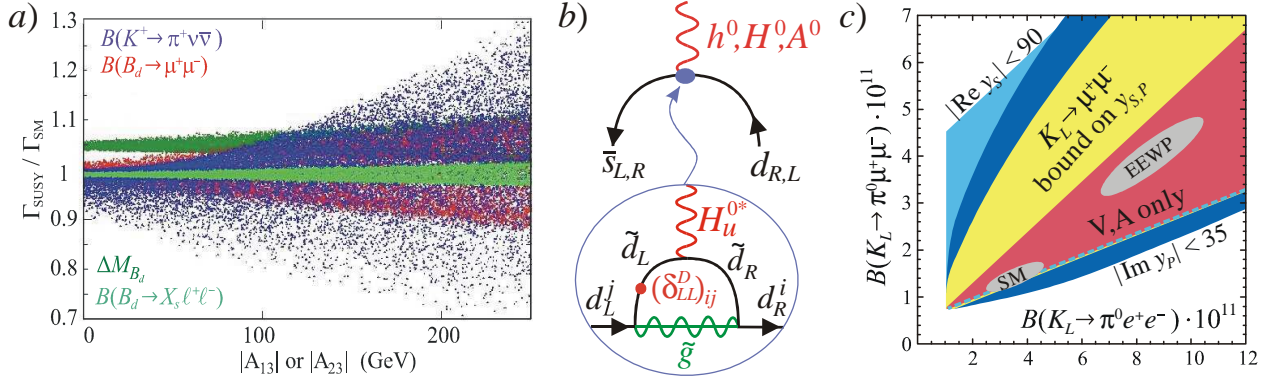


Fig. 2. a) Sensitivity of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ to \mathbf{A}^U terms, compared to B -physics observables. b) Schematic representation of the neutral Higgs FCNC beyond MFV, at large $\tan \beta$. c) Impacts of dim-6 FCNC operators in the $B(K_L \rightarrow \pi^0 \mu^+ \mu^-)$ vs. $B(K_L \rightarrow \pi^0 e^+ e^-)$ plane.

4 Supersymmetric effects in $K \rightarrow \pi \nu \bar{\nu}$

SUSY effects in the (axial-)vector operators, $(\bar{s}d)_{V \pm A}(\bar{\nu}\nu)_{V-A}$, cannot be distinguished since only $(\bar{s}d)_V(\bar{\nu}\nu)_{V-A}$ contributes to the $K \rightarrow \pi \nu \bar{\nu}$ matrix-element. All MSSM effects are thus encoded into a single complex number, $X^\nu \equiv y_L^\nu + y_R^\nu$ [19]:

$$\mathcal{H}_{eff} = y_L^\nu (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A} + y_R^\nu (\bar{s}d)_{V+A} (\bar{\nu}\nu)_{V-A} \\ \rightarrow (y_L^\nu + y_R^\nu) (\bar{s}d)_V (\bar{\nu}\nu)_{V-A}.$$

At moderate $\tan \beta$, chargino penguins are the dominant MSSM contributions because of their quadratic sensitivity to up-squark mass-insertions (Figs.1c, 1g). Within MFV, this means, given the m_t enhancement present in the δ_{LR}^U sector, that $K \rightarrow \pi \nu \bar{\nu}$ are particularly sensitive. Still, a significant enhancement would require a very light stop and chargino[21], mostly because of the constraint from $\Delta\rho$ [22]. Any enhancement $\gtrsim 5\%$ would thus falsify MFV if sparticles are found above $\sim 200\text{ GeV}$, and if $\tan \beta \gtrsim 5$ (to get rid of the H^\pm contribution). Turning on generic \mathbf{A}^U terms, the largest deviations arise in $K \rightarrow \pi \nu \bar{\nu}$, see Fig.2a[21]. Further, the decoupling is slower than for observables sensitive to chargino boxes like ε_K . All in all, given that $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has already been seen, how large the effect could be for $K_L \rightarrow \pi^0 \nu \bar{\nu}$? By an extensive, adaptive scanning over the MSSM parameter space, Ref.[23] has shown that the GN model-independent bound[24] can be saturated, which represents a factor ~ 30 enhancement of $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ over the SM.

At large $\tan \beta$, the chargino contributions may no longer represent the dominant effect. While the Higgs FCNC obviously does not contribute (Fig.1e), higher order effects in the H^\pm contribution to the Z -penguin (Fig.1h), sensitive to δ_{RR}^D , can become sizeable beyond MFV[25]. Further, this contribution is slowly decoupling as M_H increases compared to tree-level neutral Higgs exchanges, as for example in $B_{s,d} \rightarrow \mu^+ \mu^-$.

SUSY effects in other dimension-six operators, $(\bar{s}d)(\bar{\nu}(\mathbf{1}, \gamma_5)\nu)$ and $(\bar{s}\sigma_{\mu\nu}d)(\bar{\nu}\sigma^{\mu\nu}(\mathbf{1}, \gamma_5)\nu)$, require active right-handed neutrinos and will not be discussed here[26]. Another possible class of operators, since the neutrino flavors are not detected, are

$(\bar{s}\Gamma^A d)(\bar{\nu}^i \Gamma^B \nu^j)$ with $i \neq j$ and $\Gamma^{A,B}$ some Dirac structures. In the MSSM, such lepton-flavor violating operators arise only from suppressed box diagrams, and cannot lead to significant effects[27]. However, they could be sizeable in the presence of R-parity violating terms[27,28].

5 Supersymmetric effects in $K_L \rightarrow \pi^0 \ell^+ \ell^-$

Though the SM predictions for these modes are less accurate than for $K \rightarrow \pi \nu \bar{\nu}$, they are sensitive to more types of New Physics operators[13]. Indeed, the final-state leptons are now charged and massive. Therefore, besides electromagnetic effects, common to both the muon and electron modes, the relatively large muon mass opens the possibility to probe a whole class of helicity-suppressed effects.

SUSY effects in the QCD operators, i.e. in the chromomagnetic $\bar{s}\sigma_{\mu\nu}dG^{\mu\nu}$ or four-quark operators, have no direct impact on $K_L \rightarrow \pi^0 \ell^+ \ell^-$. Indeed, as said in Sect. 2, the two-photon CPC piece is fixed entirely in terms of the measured $K \rightarrow \pi\pi\pi, \pi\gamma\gamma$ modes[11,14], while the ICPV contribution is fixed from the measured ε_K and $K_S \rightarrow \pi^0 \ell^+ \ell^-$ rate[9]. At the low scale $\mu \lesssim m_c$, new physics can thus explicitly enter through semi-leptonic FCNC operators only.

SUSY effects in the SM operators, which are the vector and axial-vector operators, can in principle be disentangled thanks to the different sensitivities of the two modes to the axial-vector current (as discussed in Sec. 2, it also produces $\ell^+ \ell^-$ in a helicity-suppressed 0^{-+} state). Various MSSM contributions can enter in y_{7A} and y_{7V} . First, chargino contributions to the Z -penguin (Fig.1c) enter as $y_{7A}, y_{7V} \sim (\delta_{RL}^U)^*_{32} (\delta_{RL}^U)_{31}$, and are thus directly correlated to the corresponding contribution to $K \rightarrow \pi \nu \bar{\nu}$ [21,29]. Within MFV, the maximal effect for $K_L \rightarrow \pi^0 \ell^+ \ell^-$ is about one third of the one for $K_L \rightarrow \pi^0 \nu \bar{\nu}$, hence may be inaccessible due to theoretical uncertainties. Secondly, gluino contributions to the electromagnetic operator $\bar{s}\sigma_{\mu\nu}dF^{\mu\nu}$ (Fig.1d) can be absorbed into $y_{7V} \sim (\delta_{RL}^D)_{12}$. Even if directly correlated with ε'/ε , sizeable effects in $K_L \rightarrow$

Table 1. Sensitivity of rare K decays to MSSM effects, with and without MFV, and with moderate and large $\tan\beta$. The dominant contributions come from single, $(\delta_j^i)_{12}$, and/or double (e.g. $(\delta_j^i)_{32}^*(\delta_j^i)_{31}$) mass insertions (see text).

MSSM scenario	$K \rightarrow \pi\nu\bar{\nu}$	$K_L \rightarrow \pi^0\ell^+\ell^-$
MFV, $\tan\beta \approx 2$	Best sensitivity, but maximal enhancement $< 20\text{-}25\%$	Less sensitive, but precisely correlated with $K \rightarrow \pi\nu\bar{\nu}$
MFV, $\tan\beta \approx 50$	Negligible effects ?	
General, $\tan\beta \approx 2$	Best probes of δ_{LR}^U (quadratic dependence in δ_{LR}^U)	δ_{LR}^U : correlated with $K \rightarrow \pi\nu\bar{\nu}$ δ_{LR}^D : correlated with ε'/ε (but cleaner)
General, $\tan\beta \approx 50$	Good probes of δ_{RR}^D (slow decoupling as $M_H \rightarrow \infty$)	Good probes of $\delta_{RR,LL}^D$, correlated with $K_L \rightarrow \mu^+\mu^-$ (but cleaner)

$\pi^0\ell^+\ell^-$ are still possible[30]. Finally, H^\pm contributions arise at large $\tan\beta$ (Fig.1h), with $y_{7A}, y_{7V} \sim (\delta_{RR}^D)_{12}$, and are directly correlated with those for $K \rightarrow \pi\nu\bar{\nu}$ [25].

SUSY effects in the (pseudo-)scalar operators, which can be helicity-suppressed or not:

$$\mathcal{H}_{eff} = y_S (\bar{s}d) (\bar{\ell}\ell) + y_P (\bar{s}d) (\bar{\ell}\gamma_5\ell) + y'_S (\bar{s}\gamma_5 d) (\bar{\ell}\ell) + y'_P (\bar{s}\gamma_5 d) (\bar{\ell}\gamma_5\ell) .$$

The first (last) two operators contribute to $K_L \rightarrow \pi^0\ell^+\ell^-$ ($K_L \rightarrow \ell^+\ell^-$). In the MSSM at large $\tan\beta$, they arise from Higgs FCNC[31], and are thus helicity-suppressed (Fig.2b). Sizeable effects for the muon mode are possible beyond MFV, where they are sensitive to $(\delta_{RR,LL}^D)_{12}$ and $(\delta_{RR}^D)_{23}(\delta_{LL}^D)_{31}$ mass-insertions. Also, even if this contribution is correlated to the one for $K_L \rightarrow \mu^+\mu^-$, given the large theoretical uncertainties for this mode, a factor ~ 4 enhancement is still allowed (Fig.2c)[13]. On the other hand, helicity-allowed contributions to these operators do not arise in the MSSM. They could appear in the presence of R-parity violating couplings, but, barring fine-tuning, their effects must be small to avoid overproducing $K_L \rightarrow e^+e^-$ [13].

SUSY effects in the (pseudo-)tensor operators, $(\bar{s}\sigma_{\mu\nu}d)(\bar{\ell}\sigma^{\mu\nu}(\mathbf{1}, \gamma_5)\ell)$, the last possible dimension six semi-leptonic FCNC operators, are helicity-suppressed in the MSSM[32] and, being also phase-space suppressed, do not lead to any significant effect [13]. Further, they cannot arise from R-parity violating couplings.

6 Conclusion

The $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $K_L \rightarrow \pi^0\nu\bar{\nu}$, $K_L \rightarrow \pi^0e^+e^-$ and $K_L \rightarrow \pi^0\mu^+\mu^-$ decay modes are the only theoretically clean windows into the $\Delta S = 1$ sector. If SUSY is discovered, the pattern of deviations they could exhibit with respect to the SM (see Table 1) will be essential to constrain the MSSM parameter-space, and hopefully unveil the nature of the SUSY-breaking mechanism.

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